Section 1: Calculus

- Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers;
- Double and Triple integrals and their applications to area, volume and surface area; Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Section 2: Linear Algebra

- Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.
- Algebra of matrices , rank and nullity; systems of linear equations, characteristic polynomial, eigenvalues and eigenvectors, diagonalization, minimal polynomial
- Cayley-Hamilton Theorem, Finite dimensional inner product spaces, orthonormal basis, Gram-Schmidt orthonormalization process, symmetric, skew-symmetric
- Hermitian, skew-Hermitian, normal, orthogonal and unitary matrices; diagonalization by a unitary matrix, Jordan canonical form; bilinear and quadratic forms, reduction and classification of quadratic forms

Section 3: Real Analysis

- Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum.
- Sequences and series, convergence, limsup, liminf.
- Bolzano Weierstrass theorem, Heine Borel theorem.
- Continuity, uniform continuity, differentiability, mean value theorem.
- Riemann sums and Riemann integral, Improper Integrals.
- Monotonic functions, types of discontinuity, functions of bounded variation.
- Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems.

- Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence, Ascoli-Arzela theorem;Weierstrass approximation theorem; contraction mapping principle
- Lebesgue measure on the real line, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Section 4: Complex Analysis

- Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.
- Functions of a complex variable: continuity, differentiability, analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula
- Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Schwarz lemma, Open mapping theorem.
- Taylor series, Laurent series, Residue theorem and applications for evaluating real integrals;.
- Rouche's theorem, Argument principle, Schwarz lemma; Conformal mappings, Mobius transformations.

Section 5: Ordinary Differential equations

- First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients, singular solutions of first order ODEs.
- Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations
- Sturm's oscillation and separation theorems, Sturm-Liouville eigenvalue problems, Planar autonomous systems of ordinary differential equations: Stability of stationary points for linear systems

with constant coefficients, Linearized stability, Lyapunov functions, Green's function
- General theory of homogenous and non-homogeneous linear ODEs, variation of parameters,

Section 6: Algebra

- Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Group action, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Cayley's theorem, class equations, Sylow theorems
- Principle ideal domains, Euclidean domains, polynomial rings, Eisenstein's irreducibility criterion; Fields, finite fields, field extensions, algebraic extensions, algebraically closed fields, Galois Theory.
- Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements.
- Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø- function, primitive roots.

Section 7: Functional Analysis

- Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces
- Hilbert spaces, orthonormal bases, projection theorem,Riesz representation theorem, spectral theorem for compact self-adjoint operators.

Section 8: Numerical Analysis

- Systems of linear equations: Direct methods (Gaussian elimination, LU decomposition, Cholesky factorization), Iterative methods (Gauss-Seidel and Jacobi) and their convergence for diagonally dominant coefficient matrices; Numerical solutions of nonlinear equations: bisection method, secant method, Newton-Raphson method, fixed point iteration;
- Finite differences, Interpolation, Lagrange, Hermite and spline interpolation

- Lagrange and Newton forms of interpolating polynomial, Error in polynomial interpolation of a function; Numerical differentiation and error.
- Numerical integration: Trapezoidal and Simpson rules, Newton-Cotes integration formulas, composite rules, mathematical errors involved in numerical integration formulae; Numerical solution of initial value problems for ordinary differential equations: Methods of Euler, Runge-Kutta method of order 2.

Section 9: Partial Differential Equations

- Method of characteristics for first order linear and quasilinear partial differential equations; Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs.
- Second order partial differential equations in two independent variables: classification and canonical forms, method of separation of variables for Laplace equation in Cartesian and polar coordinates, heat and wave equations in one space variable
- Wave equation: Cauchy problem and d'Alembert formula, domains of dependence and influence, non-homogeneous wave equation; Heat equation: Cauchy problem; Laplace and Fourier transform methods.

Section 10: Topology

- Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn’s Lemma.

Section 11: Linear Programming

- Linear programming models, convex sets, extreme points;Basic feasible solution,graphical method, simplex method, two phase methods, revised simplex method ; Infeasible and unbounded linear programming models, alternate optima; Duality theory, weak duality and strong duality; Balanced and unbalanced transportation problems
- Initial basic feasible solution of balanced transportation problems (least cost method, north-west corner rule, Vogel’s approximation method); Optimal solution, modified distribution method; Solving assignment problems, Hungarian method.
- Elementary queuing and inventory models.

- Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.

Calculus of Variations:

- Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema.
- Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations:

- Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Classical Mechanics:

- Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

Statistics and probability theory

- Descriptive statistics, exploratory data analysis
- Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions.
- Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems (i.i.d. case).
- Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.
- Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.
- Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests,

likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests.

- Simple nonparametric tests for one and two sample problems, rank correlation and test for independence.
- Elementary Bayesian inference.
- Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models.
- Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression.
- Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction techniques: Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.
- Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling. Ratio and regression methods.
- Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2K factorial experiments: confounding and construction.
- Hazard function and failure rates, censoring and life testing, series and parallel systems.